Nonlinear Saturation Controller for Suppressing Inclined Beam Vibrations

Usama H. Hegazy *, Noura A Salem

Abstract—In this paper, we present the numerical and perturbation solutions of an inclined beam to external and parametric forces with two different controllers, positive position feedback (PPF) and nonlinear saturation (NS) controllers and found that the (NS) one is an effective controller. The frequency response function and the phase plane methods are used to investigate the system behavior and its stability. All possible resonance cases will be extracted and effect of different parameters on system behavior at resonance are studied.

Keywords: Beams, Vibration control, Perturbation

1 Introduction

Vibrations are the cause of discomfort, disturbance, damage, and sometimes destruction of machines and structures. It must be reduced or controlled or eliminated. One of the most common methods of vibration control is the dynamic absorber. It has the advantages of low cost and simple operation at one model frequency. In the domain of many mechanical vibration systems the coupled nonlinear vibration of such systems can be reduced to nonlinear second order differential equations which are solved analytically and numerically.

Elhefnawy and Bassiouny [1] studied the nonlinear instability problem of two superposed dielectric fluids by using the method of multiple scales. Frequency response curves are presented graphically. The stability of the proposed solution is determined. Numerical solutions are presented graphically for the effects of the different parameters on the system stability, response and chaos. El Behady and El-Zahar [2] studied the effect of the nonlinear controller on the vibrating system. The approximate solutions up to the second order are derived using the method of multiple scales perturbation technique near the primary, principal parametric and internal resonance cases. Moreover, they investigated the stability of the solution using both phase plane method and frequency response equations, and the effects of different parameters on the vibration of the system. Warminski et al. [3] studied active suppression of nonlinear composite beam vibrations by selected control algorithms. Jun et al. [4,5] extensively studied theoretical and experimental research on the saturation phenomenon. Eissa et al. [6,7] investigated a single-degree-of-freedom nonlinear oscillating system subject to multi-parametric and/or external excitations. The multiple time scale perturbation technique is applied to obtain solution up to the third order approximation to extract and study the available resonance cases. They reported the occurrence of saturation phenomena at different parameters values. Kwak and Heo [8] presented effectiveness of the PPF algorithm applied for a model of a solar panel, where the first four modes of vibration have been considered. Siewe and Hegazy [9] applied different active controllers to suppress the vibration of a micromechanical resonator system. Moreover, a time-varying stiffness is introduced to control the chaotic motion of the considered system. Eissa and Amer [10] and Yaman and Sen [11] studied the vibration control of a cantilever beam subject to both external and parametric excitations but with different controllers. Golnaraghi [12] indicated that when the system is excited at a frequency near the high natural frequency, the structure responds at the frequency of the excitation and the amplitude of the response increases with the excitation amplitude. Oueini et al. [13] proposed a nonlinear control law, which is based on cubic velocity feedback, to suppress the vibrations of the first mode of a cantilever beam when subjected to a principal parametric excitation. The method of multiple scales is used to derive two first-order differential equations governing the time evolution of the amplitude and phase of the response. Then, a bifurcation analysis is conducted to examine the stability of the closed-loop system and to investigate the performance of the control law. The theoretical and experimental findings indicate that the control law leads to effective vibration suppression and bifurcation control. El-Serafi et al. [14,15] showed how effective is the active control on vibration reduction of different modes of motion at resonance. They demonstrated the advantages of active control over the passive one. Hegazy [16] studied the nonlinear dynamics and vibration control of an electromechanical seismograph system with time-varying stiffness. An active control method is applied to the system based on cubic velocity feedback. In [17], Hegazy investigated The problem of suppressing the vibrations of a hinged–hinged flexible beam that is subjected to primary and principal parametric excitations. Different control laws are proposed, and saturation phenomenon is investigated to suppress the vibrations of the system. El-Ganaini et al. [18] applied positive position feedback active controller to suppress the vibration of a nonlinear system when subjected to external primary resonance excitation. The multiple scale perturbation method is applied to obtain a first-

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order approximate solution. The equilibrium curves for various controller parameters are plotted. The stability of the steady state solution is investigated using frequency response equations. The approximate solution is numerically verified. They found that all predictions from analytical solutions are in good agreement with the numerical simulations.

2 System model

The modified second-order nonlinear ordinary differential equation that describes the motion of the inclined beam is given by [11]

$$u'' + \mu_1 u' + \omega_s^2 u + \beta_1 u^3 + \beta_2 u^5 - \delta\left(uu'^2 + u^2 u''\right) = f_1 \cos(\Omega t)\cos(\alpha) + uf_2 \cos(\Omega t)\sin(\alpha) + \tau F_c(t). \quad (1)$$

Where u, u' and u'' represent displacement, velocity and acceleration of the vibrating beam, respectively, ω_s is the natural frequency, μ_1 is the damping coefficient, β_1, β_2 and δ are nonlinear coefficients, f_1 and f_2 are the external and parametric forcing amplitudes, respectively, Ω is the excitation frequency, α is the orientation angle , τ is the gain and $F_c(t)$ is the control signal.

We introduce a two second-order nonlinear controllers, which are coupled to the main system through a control law. Then, the equation governing the dynamics of the controllers is suggested as

$$v'' + 2\xi\omega_c v' + \omega_c^2 v = \rho F_f(t) \quad (2)$$

where v, v' and v'' represent displacement, velocity and acceleration of the controller, ω_c is the natural frequency, ζ is the damping coefficient and ρ is the gain. We choose the control signal $F_c = v$ and feedback signal $F_f = u$ for (PPF) control, and $F_f = v^2$, $F_f = uv$ for (NS) control. So the closed loop system equations to the both controllers are:

(i) Positive Position Feedback (PPF) control

$$u'' + \mu_1 u' + \omega_s^2 u + \beta_1 u^3 + \beta_2 u^5 - \delta\left(uu'^2 + u^2 u''\right) = f_1 \cos(\Omega t)\cos(\alpha) + uf_2 \cos(\Omega t)\sin(\alpha) + \tau v, \quad (3)$$

$$v'' + 2\xi\omega_c v' + \omega_c^2 v = \rho u \quad (4)$$

(ii) Nonlinear Saturation (NS) control

$$u'' + \mu_1 u' + \omega_s^2 u + \beta_1 u^3 + \beta_2 u^5 - \delta\left(uu'^2 + u^2 u''\right) = f_1 \cos(\Omega t)\cos(\alpha) + uf_2 \cos(\Omega t)\sin(\alpha) + \tau v^2, \quad (5)$$

$$v'' + 2\xi\omega_c v' + \omega_c^2 v = \rho uv \quad (6)$$

3 Numerical Integration

The numerical study of the response and the stability of two nonlinear systems, are conducted. Each system is represented by two (the plant and the absorber) coupled second order nonlinear differential equations. The plant (oriented beam) has quadratic, cubic and quintic nonlinearities and is subjected to external and parametric excitations. The coupling terms are either produce the positive position absorber or nonlinear sink absorber. All possible resonance cases are extracted and effects of different parameters and controllers on the plant are discussed and reported.

3.1 Time-response solution

The time response of the nonlinear systems (3), (4) and (5), (6) has been investigated applying fourth order Runge-Kutta numerical method and the results are shown in Figs. (1) and (2), respectively. The phase plane method is used to give an indication about the stability of the system. Figs. (1a) and (1b) show the non-resonant behavior of the main system and the PPF absorber, respectively, with fine limit cycle for the plant. Whereas, a chaotic behavior is illustrated in Figs.(1c) and (1d) for both the plant and the absorber at the simultaneous primary resonance case. Fig. (2) show the responses of the plant and the NS absorber at non-resonance, Figs. (2a) and (2b), respectively and at two resonance cases, Figs. (2c) and (2d). It is clear that the response of the plant with the NS absorber is much better than of PPF absorber. The NS might be more effective in controlling the behavior of the main system at resonance, which resulted in a slight chaotic resonant response, Fig. (2c) or a modulated amplitude, Fig. (2e). Therefore, the NS absorber will be considered and coupled with the main system for further investigation in the following section.

4 Multiple-Time scales analysis

The nonlinear differential equation (5) with NS controller (6) is scaled using the perturbation parameter ε as follows

$$u'' + \varepsilon\mu_1 u' + \omega_s^2 u + \varepsilon\beta_1 u^3 + \varepsilon\beta_2 u^5 - \varepsilon\delta\left(uu'^2 + u^2 u''\right) = \varepsilon f_1 \cos(\Omega t)\cos(\alpha) + \varepsilon uf_2 \cos(\Omega t)\sin(\alpha) + \varepsilon\tau v^2, \quad (5a)$$

$$v'' + 2\xi\varepsilon\omega_c v' + \omega_c^2 v = \varepsilon\rho uv. \quad (6a)$$

Applying the multiple scales method, we obtain first order approximate solutions for equation (3) and (4) by seeking the solutions in the form

$$u(T_0,T_1)=u_0(T_0,T_1)+\varepsilon u_1(T_0,T_1),$$
$$v(T_0,T_1)=v_0(T_0,T_1)+\varepsilon v_1(T_0,T_1). \quad (7)$$

where ε is a small dimensionless book keeping perturbation parameter, $T_0=t$ and $T_1=\varepsilon T_0$ are the fast and slow time scales, respectively. The time derivatives transform is recast in terms of the new time scales as

$$\frac{d}{dt}=D_0+\varepsilon D_1, \frac{d^2}{dt^2}=D_0^2+2\varepsilon D_0 D_1 \quad (8)$$

where

$$D_0=\frac{\partial}{\partial T_0} \text{ and } D_1=\frac{\partial}{\partial T_1}. \quad (9)$$

Substituting u and time derivatives from equations (7) and (8), we get

$$u=u_0+\varepsilon u_1,$$
$$u'=D_0u_0+\varepsilon D_0u_1+\varepsilon D_1u_0+\varepsilon^2 D_1u_1, \quad (10)$$
$$u''=D_0{}^2u_0+\varepsilon D_0{}^2u_1+2\varepsilon D_0D_1u_0+2\varepsilon^2 D_0D_1u_1,$$

and

$$v=v_0+\varepsilon v_1,$$
$$v'=D_0v_0+\varepsilon D_0v_1+\varepsilon D_1v_0+\varepsilon^2 D_1v_1, \quad (11)$$
$$v''=D_0{}^2v_0+\varepsilon D_0{}^2v_1+2\varepsilon D_0D_1v_0+2\varepsilon^2 D_0D_1v_1.$$

Substituting equations (10) and (11) into equations (5a) and (6a), we get

$$D_0^2u_0+\varepsilon D_0^2u_1+2\varepsilon D_0D_1u_0+\varepsilon\mu_1D_0u_0$$
$$+\omega_s^2u_0+\varepsilon\omega_s^2u_1+\varepsilon\beta_1u_0^3+\varepsilon\beta_2u_0^5-2\varepsilon\delta D_0^2u_0^3 \quad (12)$$
$$-\varepsilon f_1\cos(\Omega t)\cos(\alpha)-\varepsilon u_0f_2\cos(\Omega t)\sin(\alpha)-\varepsilon\tau v_0^2=0,$$

and

$$D_0^2v_0+\varepsilon D_0^2v_1+2\varepsilon D_0D_1v_0+2\varepsilon\xi\omega_cD_0v_0$$
$$+\omega_c^2v_0+\varepsilon\omega_c^2v_1-\varepsilon\rho u_0v_0=0. \quad (13)$$

Equating the coefficient of same powers of ε in equations (12) and (13), gives

$O(\varepsilon^0)$:

$$\left(D_0^2+\omega_s^2\right)u_0=0, \quad (14)$$

and

$$\left(D_0^2+\omega_c^2\right)v_0=0. \quad (15)$$

$O(\varepsilon^1)$:

$$\left(D_0^2+\omega_s^2\right)u_1=-2D_0D_1u_0-\mu_1D_0u_0-\beta_1u_0^3-\beta_2u_0^5$$
$$+2\delta D_0^2u_0^3+f_1\cos(\Omega t)\cos(\alpha) \quad (16)$$
$$+u_0f_2\cos(\Omega t)\sin(\alpha)+\tau v_0^2,$$

and

$$\left(D_0^2+\omega_c^2\right)v_1=-2D_0D_1v_0-2\varepsilon\xi\omega_cD_0v_0+\rho u_0v_0. \quad (17)$$

The general solution of equations (14) and (15) is given by

$$u_0=A(T_1)e^{i\omega_sT_0}+\bar{A}(T_1)e^{-i\omega_sT_0}, \quad (18)$$

and

$$v_0=B(T_1)e^{i\omega_cT_0}+\bar{B}(T_1)e^{-i\omega_cT_0}. \quad (19)$$

where the quantities $A(T_1)$ and $B(T_1)$ are unknown function in T_1. Now to solve equations (16) and (17), we substitute equations (18) and (19) into them, then using the forms

$$\cos(\Omega T_0)=\frac{e^{i\Omega T_0}+e^{-i\Omega T_0}}{2} \text{ and } \sin(\Omega T_0)=\frac{e^{i\Omega T_0}-e^{-i\Omega T_0}}{2i}$$

we obtain

$$\left(D_0^2+\omega_s^2\right)u_1=\left(-2i\omega_sA'-\mu_1i\omega_sA-3\beta_1A^2\bar{A}\right.$$
$$\left.-10\beta_2A^3\bar{A}^2-6\omega_s^2\delta A^2\bar{A}\right)e^{i\omega_sT_0}$$
$$+\left(-\beta_1A^3-5\beta_2A^4\bar{A}-18\omega_s^2\delta A^3\right)e^{3i\omega_sT_0}$$
$$-\beta_2A^5e^{5i\omega_sT_0}+\frac{1}{2}f_1e^{i\Omega T_0}\cos(\alpha) \quad (20)$$
$$+\frac{1}{2}f_2Ae^{i(\omega_s+\Omega)T_0}\sin(\alpha)+\frac{1}{2}f_2Ae^{i(\omega_s-\Omega)T_0}\sin(\alpha)$$
$$+\tau B^2e^{2i\omega_cT_0}+\tau B\bar{B}+cc,$$

and

$$\left(D_0^2+\omega_c^2\right)v_1=\left(-2i\omega_cB'-2i\xi\omega_c^2B\right)e^{i\omega_cT_0}$$
$$+\rho BAe^{i(\omega_s+\omega_c)T_0}+\rho\bar{A}Be^{i(\omega_c-\omega_s)T_0}+cc. \quad (21)$$

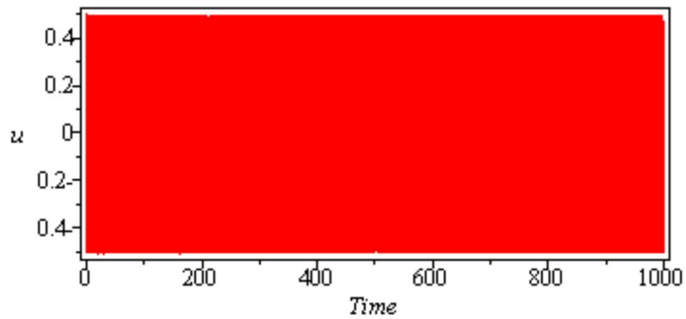
where cc denotes the complex conjugate terms.

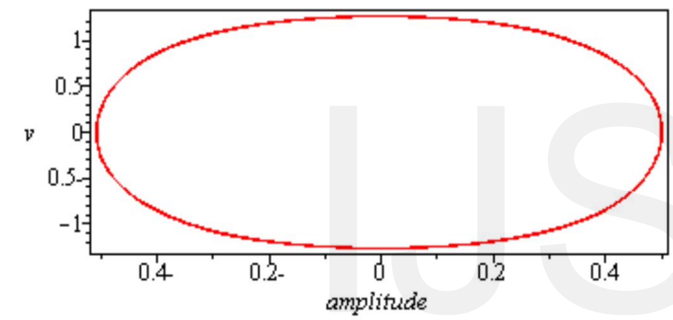
The particular solution of equations (20) and (21) can be written in the following form

$$u_1(T_0,T_1)=$$
$$A_1e^{i\omega_sT_0}-\frac{1}{8\omega_s^2}\left(-\beta_1A^3-5\beta_2A^4\bar{A}-18\omega_s^2\delta A^3\right)e^{3i\omega_sT_0}$$
$$+\frac{1}{24\omega_s^2}\beta_2A^5e^{5i\omega_sT_0}+\frac{1}{2(\omega_s-\Omega)(\omega_s+\Omega)}f_1\cos(\alpha)e^{i\Omega T_0}$$
$$-\frac{1}{2\Omega(2\omega_s+\Omega)}f_2Ae^{i(\omega_s+\Omega)T_0}\sin(\alpha)$$
$$+\frac{1}{2\Omega(2\omega_s-\Omega)}f_2Ae^{i(\omega_s-\Omega)T_0}\sin(\alpha) \quad (22)$$
$$+\frac{1}{(-2\omega_c+\omega_s)(2\omega_c+\omega_s)}\tau B^2e^{2i\omega_cT_0}+\tau B\bar{B}+cc,$$

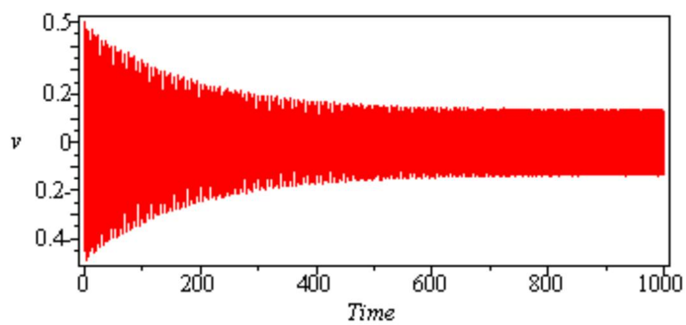
and

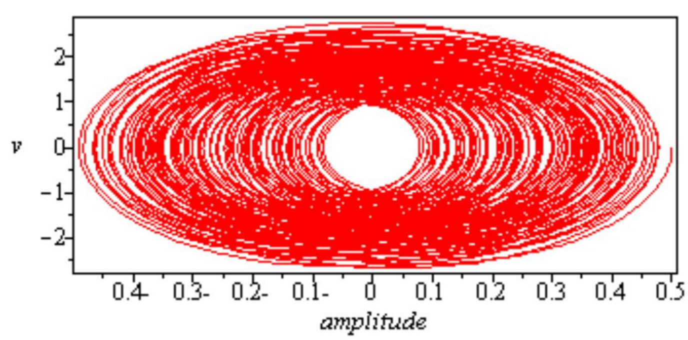
$$v_1(T_0,T_1) = B_1 e^{i\omega_c T_0} - \frac{1}{\omega_s\left(\omega_s + 2\omega_c\right)}\rho BA e^{i(\omega_s+\omega_c)T_0} - \frac{1}{\omega_s\left(\omega_s - 2\omega_c\right)}\rho \bar{A} B e^{i(\omega_c-\omega_s)T_0} + cc. \quad (23)$$



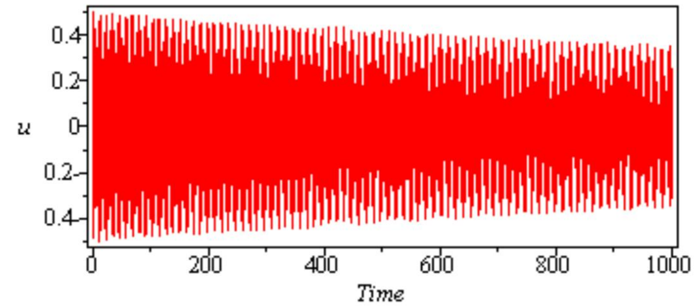


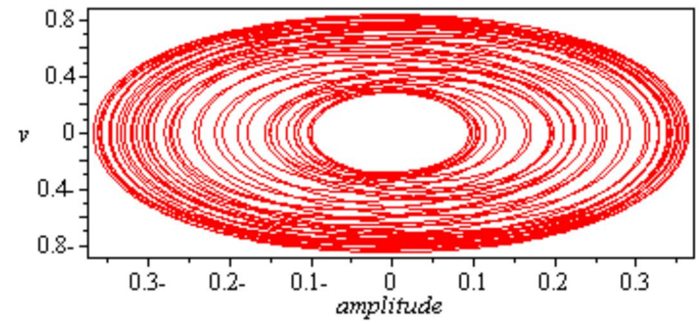
(a) Non-resonant time series of the plant



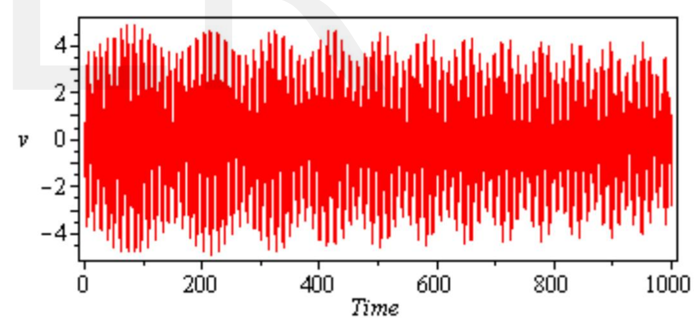


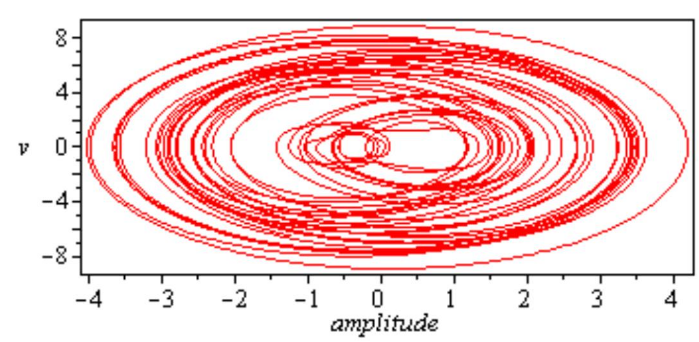
(b) Non-resonant time series of the controller





(c) Resonant time series of the plant when $\Omega = \omega_s$ and $\omega_s = \omega_c$





(d) Resonant time series of the controller when $\Omega = \omega_s$ and $\omega_s = \omega_c$

Fig. 1 Non-resonant and resonant time history solution of the plant and the (PPF) controller when:

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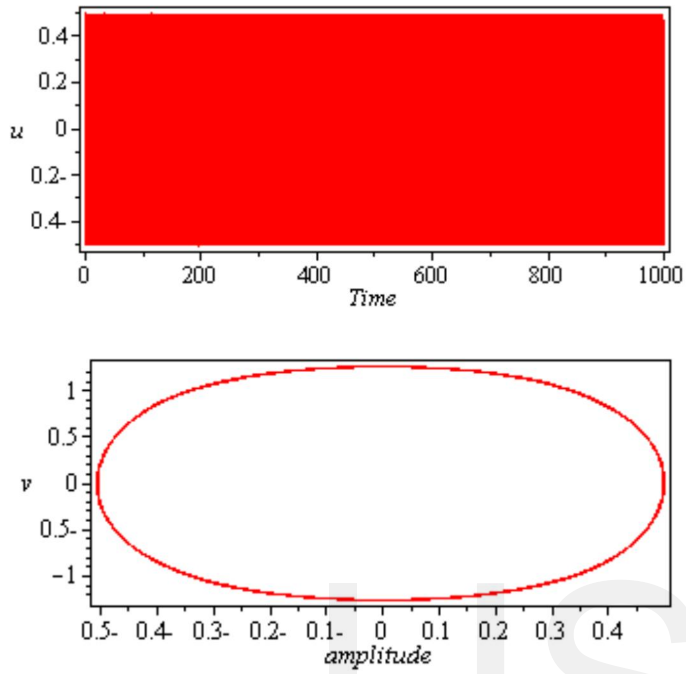
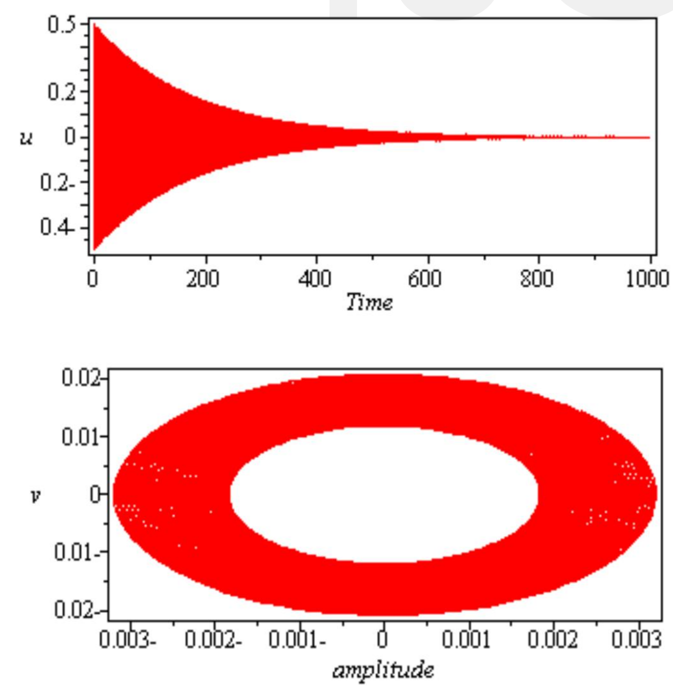
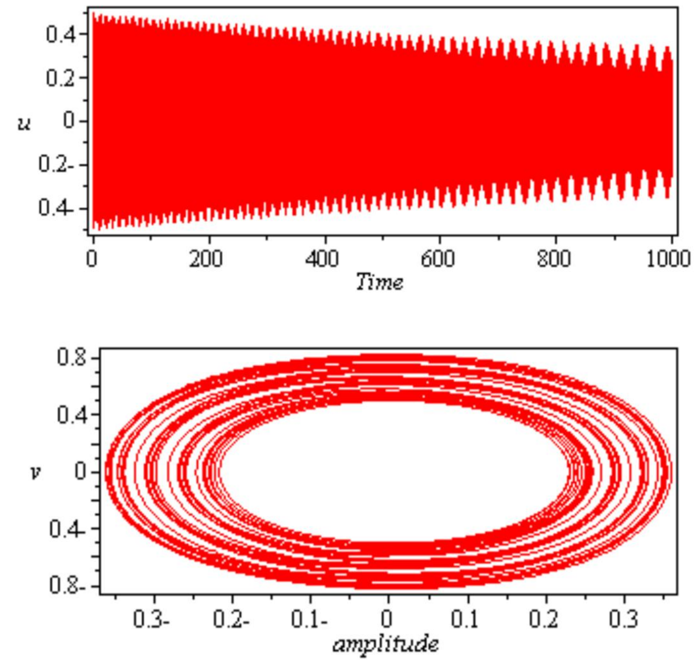
$\omega_s = 2.1,\ \beta_1 = 15.0,\ \delta = 0.03,\ \mu_1 = 0.0005,\ \Omega = 2.7,$
$\beta_2 = 5.0,\ f_1 = 0.4,\ f_2 = 0.2,\ \alpha = 30^0,\ \tau = 0.1,$
$\xi = 0.0001,\ \rho = 10.0,\ \omega_c = 6.5.$



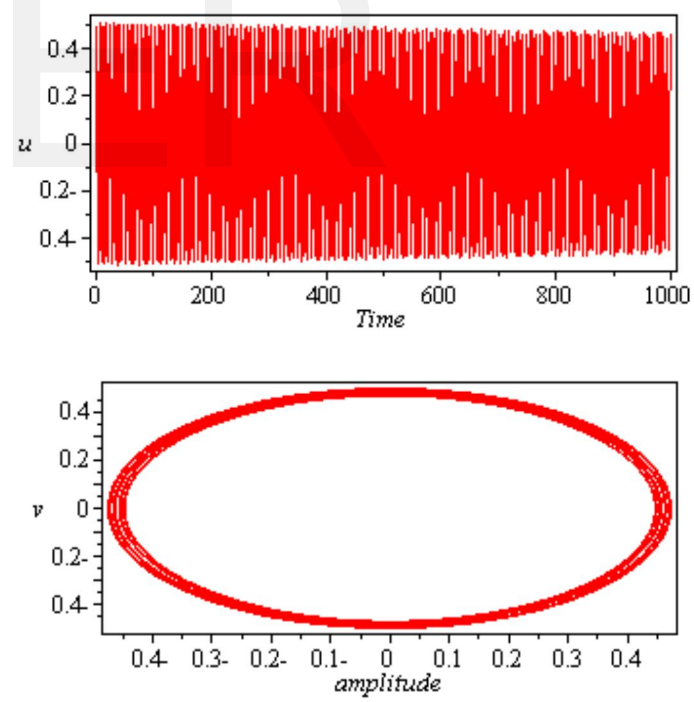
(a) Non-resonant time series of the plant



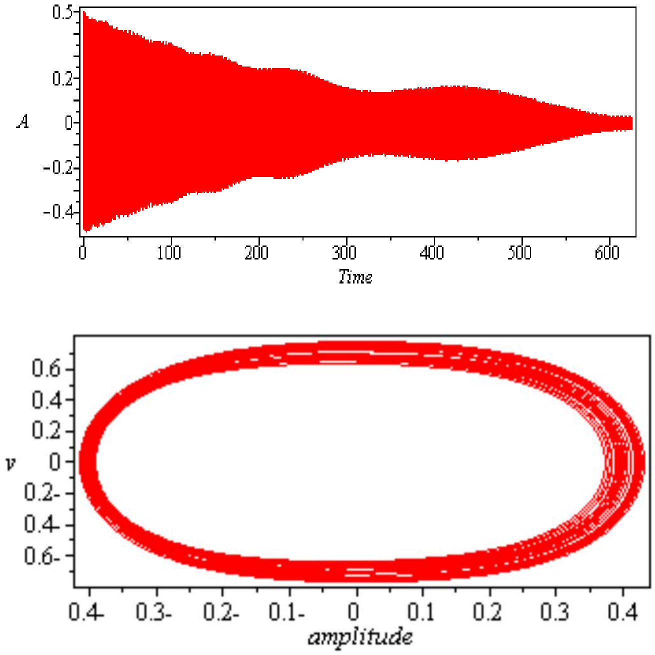
(b) Non-resonant time series of the controller



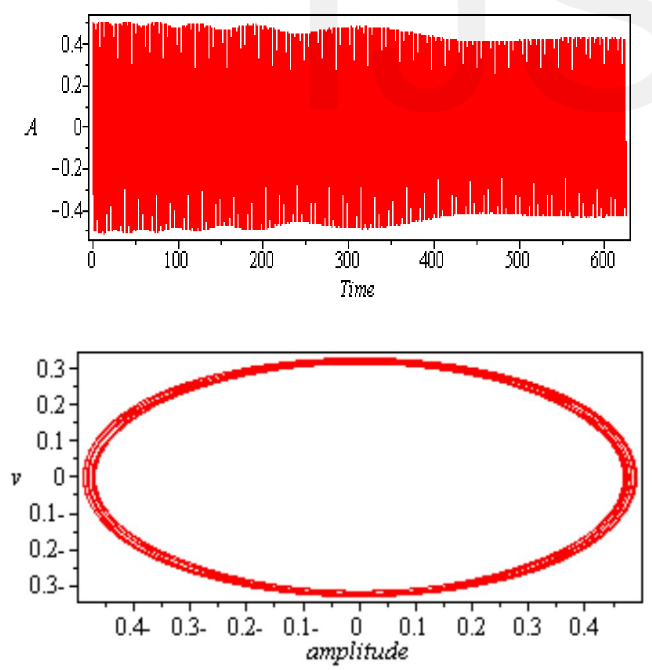
(c) Resonant time series of the plant when $\Omega = \omega_c$ and $\omega_c = \frac{1}{2}\omega_s$



(d) Resonant time series of the controller when $\Omega = \omega_c$ and $\omega_c = \frac{1}{2}\omega_s$



(e) Resonant time series of the plant when $\Omega = 2\omega_s$ and $\omega_c = \frac{1}{2}\omega_s$



(f) Resonant time series of the controller when $\Omega = 2\omega_s$ and $\omega_c = \frac{1}{2}\omega_s$

Fig. 2 Non-resonant and resonant time history solution of the plant and the (NS) controller system when:

$$\omega_s = 2.1,\ \beta_1 = 15.0,\ \delta = 0.03,\ \mu_1 = 0.0005,\ \Omega = 2.7,$$
$$\beta_2 = 5.0,\ f_1 = 0.4,\ f_2 = 0.2,\ \alpha = 30^0,\ \tau = 0.1,$$
$$\xi = 0.0001,\ \rho = 0.1,\ \omega_c = 6.5.$$

5 Stability Analysis

We shall investigate the stability of the system at the simultaneous resonance condition $\Omega = \omega_c$ and $\omega_c = \frac{1}{2}\omega_s$.

In this case we introduce the detuning parameters σ_1 and σ_2 such that

$$\Omega = \omega_c + \varepsilon\sigma_1 \text{ and } \omega_c = \frac{1}{2}\omega_s + \varepsilon\sigma_2 \tag{24}$$

Substituting equation (24) into equations (20) and (21), eliminating the terms that produce secular term and performing some algebraic manipulations, we obtain

$$-2i\,\omega_s A' - \mu_1 i\,\omega_s A - 3\beta_1 A^2\bar{A} - 10\beta_2 A^3\bar{A}^2 - 6\omega_s^2\delta A^2\bar{A}$$
$$+\frac{1}{2}f_1 e^{i\sigma_1 T_1}\cos(\alpha) + \tau B^2 e^{i\,2\sigma_2 T_1} = 0, \tag{25}$$

and

$$\left(-2i\omega_c B' - 2i\xi\omega_c^2 B\right) + \rho A\bar{B}e^{-2i\sigma_2 T_1} = 0. \tag{26}$$

Substituting $A = \frac{1}{2}a_1 e^{i\theta_1}$ and $B = \frac{1}{2}a_2 e^{i\theta 2}$, we obtain the following equations that describe the modulations of amplitudes and phases of the motions

$$2a_1' = -\mu_1 a_1 + \frac{1}{\omega_s}f_1\sin\left(-\theta_1 + \sigma_1 T_1\right)\cos(\alpha)$$
$$+\frac{1}{2\omega_s}\tau a_2^2\sin\left(-\theta_1 + 2\theta_2 + 2\sigma_2 T_1\right), \tag{27}$$

$$2a_1\theta_1' - \frac{3}{4\omega_s}\beta_1 a_1^3 - \frac{5}{8\omega_s}\beta_2 a_1^5 - \frac{3}{2}\omega_s\delta a_1^3$$
$$+\frac{1}{\omega_s}f_1\cos\left(-\theta_1 + \sigma_1 T_1\right)\cos(\alpha) \tag{28}$$
$$+\frac{1}{2\omega_s}\tau a_2^2\cos\left(-\theta_1 + 2\theta_2 + \sigma_2 T_1\right) = 0.$$

and

$$a_2' = -\xi\omega_c a_2 - \frac{1}{4\omega_c}\rho a_1 a_2\sin\left(-\theta_1 + 2\theta_2 + 2\sigma_2 T_1\right), \tag{29}$$

$$a_2\theta_2' + \frac{1}{4\omega_c}\rho a_1 a_2\cos\left(-\theta_1 + 2\theta_2 + 2\sigma_2 T_1\right) = 0. \tag{30}$$

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Let $\Lambda_1 = \frac{1}{\omega_s} f_1$, $\Lambda_2 = \frac{1}{2\omega_s} \tau a_2^2$, $\Lambda_3 = \frac{1}{4\omega_c} \rho a_1 a_2$

$\gamma_1 = (-\theta_1 + \sigma_1 T_1)$ and $\gamma_2 = (-\theta_1 + 2\theta_2 + 2\sigma_2 T_1)$

Then, equations (27) - (30) become

$$2a_1' = -\mu_1 a_1 + \Lambda_1 \sin(\gamma_1)\cos(\alpha) + \Lambda_2 \sin(\gamma_2), \tag{31}$$

$$2a_1\gamma_1' = 2\sigma_1 a_1 - \frac{3}{4\omega_s}\beta_1 a_1^3 - \frac{5}{8\omega_s}\beta_2 a_1^5 - \frac{3}{2}\omega_s \delta a_1^3 + \Lambda_1 \cos(\gamma_1)\cos(\alpha) + \Lambda_2 \cos(\gamma_2). \tag{32}$$

and

$$a_2' = -\xi\omega_c a_2 - \Lambda_3 \sin(\gamma_2), \tag{33}$$

$$\frac{1}{2}a_2(\gamma_1' - \gamma_2') = \frac{1}{2}a_2(\sigma_1 - 2\sigma_2) + \Lambda_3 \cos(\gamma_2). \tag{34}$$

The steady state solutions correspond to constant a_1, a_2 and γ_1, γ_2 that is $a_1' = a_2' = \gamma_1' = \gamma_2' = 0$. Thus we get

$$\mu_1 a_1 = \Lambda_1 \sin(\gamma_1)\cos(\alpha) + \Lambda_2 \sin(\gamma_2), \tag{35}$$

$$-2\sigma_1 a_1 + \frac{3}{4\omega_s}\beta_1 a_1^3 + \frac{5}{8\omega_s}\beta_2 a_1^5 + \frac{3}{2}\omega_s \delta a_1^3 = \Lambda_1 \cos(\gamma_1)\cos(\alpha) + \Lambda_2 \cos(\gamma_2). \tag{36}$$

and

$$\xi\omega_c a_2 = -\Lambda_3 \sin(\gamma_2), \tag{37}$$

$$-\frac{1}{2}a_2(\sigma_1 - 2\sigma_2) = \Lambda_3 \cos(\gamma_2). \tag{38}$$

From equations (35) - (38), we have

$$(\mu_1 a_1)^2 + \left(-2\sigma_1 a_1 + \frac{3}{4\omega_s}\beta_1 a_1^3 + \frac{5}{8\omega_s}\beta_2 a_1^5 + \frac{3}{2}\omega_s \delta a_1^3\right)^2 = \Lambda_1^2 \cos^2(\alpha) + \Lambda_2^2 + 2\Lambda_1\Lambda_2 \cos(\alpha), \tag{39}$$

$$(\xi\omega_c a_2)^2 + \left(\frac{1}{2}a_2(\sigma_1 - 2\sigma_2)\right)^2 = \Lambda_3^2. \tag{40}$$

Equations (39) and (40) are called frequency response equations of the plant and the NS controller, respectively.

(A) TRIVIAL SOLUTION

To determine the stability of the trivial solutions, we investigate the solutions of the linearized form equations (25) and (26), that is

$$-2i\,\omega_s A' - \mu_1 i\,\omega_s A = 0, \tag{41}$$

and

$$-2i\,\omega_c B' - 2i\,\xi\omega_c^2 B = 0. \tag{42}$$

We express *A* and *B* in the following Cartesian forms

$A = \frac{1}{2}(p_1 - ip_2)e^{i\phi_1 T_1}$ and $B = \frac{1}{2}(p_3 - ip_4)e^{i\phi_2 T_1}$

where p_1, p_2, p_3, p_4 are real. We obtain

$$-2i\,\omega_s \left(\frac{1}{2}(p_1' - ip_2')e^{i\phi_1 T_1} + \frac{1}{2}i\,\phi_1(p_1 - ip_2)e^{i\phi_1 T_1}\right) - \frac{1}{2}i\,\omega_s \mu_1 (p_1 - ip_2)e^{i\phi_1 T_1} = 0. \tag{43}$$

and

$$-2i\,\omega_c \left(\frac{1}{2}(p_3' - ip_4')e^{i\phi_2 T_1} + \frac{1}{2}i\,\phi_2(p_3 - ip_4)e^{i\phi_2 T_1}\right) - i\,\xi\omega_c^2 (p_3 - ip_4)e^{i\phi_2 T_1} = 0. \tag{44}$$

Dividing both sides of equation (43) by $\omega_s e^{i\phi_1 T_1}$ and both of sides of equation (44) by $\omega_c e^{i\phi_2 T_1}$, give

$$-ip_1' - p_2' + \phi_1 p_1 - i\,\phi_1 p_2 - \frac{1}{2}ip_1\mu_1 - \frac{1}{2}p_2\mu_1 = 0. \tag{45}$$

and

$$-ip_3' - p_4' + \phi_2 p_3 - i\,\phi_2 p_4 - i\,\omega_c p_3 \xi - \omega_c p_4 \xi = 0. \tag{46}$$

Separating real and imaginary parts in equations (45) and (46) to get

$$p_1' = \left(-\frac{1}{2}\mu_1\right)p_1 + (-\phi_1)p_2, \tag{47}$$

$$p_2' = (\phi_1)p_1 + \left(-\frac{1}{2}\mu_1\right)p_2, \tag{48}$$

$$p_3' = (-\omega_c \xi)p_3 + (-\phi_2)p_4, \tag{49}$$

and

$$p_4' = (\phi_2)p_3 + (-\omega_c \xi)p_4. \tag{50}$$

Setting $J_{11} = -\frac{1}{2}\mu_1$, $J_{12} = -\phi_1$, $J_{33} = -\omega_c \zeta$, $J_{34} = -\phi_2$.

The stability of the trivial solution is investigated by evaluating the eigenvalues of the Jacobian matrix of equations (47) - (50)

$$\begin{vmatrix} J_{11} - \lambda & J_{12} & 0 & 0 \\ -J_{12} & J_{11} - \lambda & 0 & 0 \\ 0 & 0 & J_{33} - \lambda & J_{34} \\ 0 & 0 & -J_{34} & J_{33} - \lambda \end{vmatrix} = 0$$

,

which gives

$$\lambda^4+\eta_1\lambda^3+\eta_2\lambda^2+\eta_3\lambda+\eta_4=0. \tag{51}$$

where η_1,η_2,η_3 and η_4 are functions in the system parameters. According to the Routh-Hurwitz criterion the necessary and sufficient conditions for all the roots of equations (51) to have negative real parts and hence a stable solution are

$$\eta_1>0,\ \eta_1\eta_2-\eta_3>0,\ \eta_3\left(\eta_1\eta_2-\eta_3\right)-\eta_1^2\eta_4>0,\ \eta_4>0.$$

(B) NON-TRIVIAL SOLUTION

To determine the stability of the non-trivial solutions, we let $a_1=b_0+b_1(T_1),\ a_2=c_0+c_1(T_1)$ and

$$\varphi=\varphi_0+\varphi_1(T_1),\ \psi=\psi_0+\psi_1(T_1) \tag{52}$$

Substituting equation (52) into equations (31) - (34) similarly as in above, we have

$$\begin{aligned}2b_0'+2b_1'&=-\mu_1b_0-\mu_1b_1+\frac{1}{\omega_s}f_1\left(\sin\varphi_0+\varphi_1\cos\varphi_0\right)\cos(\alpha)\\&+\frac{1}{2\omega_s}\tau c_0^2\left(\sin\psi_0+\psi_1\cos\psi_0\right)+\frac{1}{2\omega_s}\tau c_0c_1\left(\sin\psi_0+\psi_1\cos\psi_0\right),\end{aligned} \tag{53}$$

$$\begin{aligned}&2b_0\varphi_0'+2b_1\varphi_0'+2b_0\varphi_1'+2b_1\varphi_1'=2\sigma_1b_0+2\sigma_1b_1\\&-\frac{3}{4\omega_s}\beta_1\left(b_0^3+3b_0^2b_1+...\right)-\frac{5}{8\omega_s}\beta_2\left(b_0^5+5b_0^4b_1+...\right)\\&-\frac{3}{2}\omega_s\delta\left(b_0^3+3b_0^2b_1+...\right)+\frac{1}{\omega_s}f_1\left(\cos\varphi_0-\varphi_1\sin\varphi_0\right)\cos(\alpha)\\&+\frac{1}{2\omega_s}\tau c_0^2\left(\cos\psi_0-\psi_1\sin\psi_0\right)+\frac{1}{2\omega_s}\tau c_0c_1\left(\cos\psi_0-\psi_1\sin\psi_0\right),\end{aligned} \tag{54}$$

$$\begin{aligned}c_0'+c_1'&=-\xi\omega_cc_0-\xi\omega_cc_1-\frac{1}{4\omega_c}\rho b_0c_0\left(\sin\psi_0+\psi_1\cos\psi_0\right)\\&-\frac{1}{4\omega_c}\rho b_1c_0\left(\sin\psi_0+\psi_1\cos\psi_0\right)-\frac{1}{4\omega_c}\rho b_0c_1\left(\sin\psi_0+\psi_1\cos\psi_0\right),\end{aligned} \tag{55}$$

and

$$\begin{aligned}&\frac{1}{2}\left(c_0\varphi_0'+c_0\varphi_1'+c_1\varphi_0'+c_1\varphi_1'-c_0\psi_0'-c_0\psi_1'-c_1\psi_0'-c_1\psi_1'\right)\\&=\frac{1}{2}c_0\left(\sigma_1-\sigma_2\right)+\frac{1}{2}c_1\left(\sigma_1-\sigma_2\right)+\frac{1}{4\omega_c}\rho b_0c_0\left(\cos\psi_0-\psi_1\sin\psi_0\right)\\&+\frac{1}{4\omega_c}\rho b_1c_0\left(\cos\psi_0-\psi_1\sin\psi_0\right)+\frac{1}{4\omega_c}\rho b_0c_1\left(\cos\psi_0-\psi_1\sin\psi_0\right).\end{aligned} \tag{56}$$

Since b_0,c_0,φ_0 and ψ_0 are solutions of equations (31) - (34), b_1,φ_1,c_1 and ψ_1 are a very small terms and $\varphi_0'+\varphi_1'=\varphi'=0$, $\psi_0'+\psi_1'=\psi'=0$ then they can be eliminated, we have

$$\begin{aligned}b_1'&=\left(-\frac{1}{2}\mu_1\right)b_1+\left(\frac{1}{2\omega_s}f_1\cos\varphi_0\cos(\alpha)\right)\varphi_1\\&+\left(\frac{1}{4\omega_s}\tau c_0\sin\psi_0\right)c_1+\left(\frac{1}{4\omega_s}\tau c_0^2\cos\psi_0\right)\psi_1,\end{aligned} \tag{57}$$

$$\begin{aligned}\varphi_1'&=\left(\frac{\sigma_1}{b_0}-\frac{9}{8\omega_s}\beta_1b_0-\frac{25}{16\omega_s}\beta_2b_0^3-\frac{9}{4}\omega_s\delta b_0\right)b_1\\&+\left(-\frac{1}{2\omega_sb_0}f_1\sin\varphi_0\cos(\alpha)\right)\varphi_1+\left(\frac{1}{2b_0\omega_s}\tau c_0\cos\psi_0\right)c_1\\&+\left(-\frac{1}{4b_0\omega_s}\tau c_0^2\sin\psi_0\right)\psi_1,\end{aligned} \tag{58}$$

$$\begin{aligned}c_1'&=\left(-\frac{1}{4\omega_c}\rho c_0\sin\psi_0\right)b_1+\left(-\xi\omega_c-\frac{1}{4\omega_c}\rho b_0\sin\psi_0\right)c_1\\&+\left(-\frac{1}{4\omega_c}\rho b_0c_0\cos\psi_0\right)\psi_1\end{aligned} \tag{59}$$

and

$$\begin{aligned}\psi_1'&=\left(\frac{\sigma_1}{b_0}+\frac{9}{8\omega_s}\beta_1b_0+\frac{25}{16\omega_s}\beta_2b_0^3+\frac{9}{4}\omega_s\delta b_0-\frac{1}{2\omega_c}\rho\cos\psi_0\right)b_1\\&+\left(\frac{1}{c_0}(\sigma_1-\sigma_2)-\frac{1}{2b_0\omega_s}\tau c_0\cos\psi_0-\frac{1}{2c_0\omega_c}\rho b_0\cos\psi_0\right)c_1\\&+\left(\frac{1}{2\omega_c}\rho b_0\sin\psi_0+\frac{1}{4b_0\omega_s}\tau c_0^2\sin\psi_0\right)\psi_1+\left(\frac{1}{2\omega_sb_0}f_1\sin\varphi_0\cos(\alpha)\right)\varphi_1.\end{aligned} \tag{60}$$

Let

$$J_{11}=-\frac{1}{2}\mu_1,\ J_{12}=\frac{1}{2\omega_s}f_1\cos\varphi_0\cos(\alpha),$$

$$,J_{13}=\frac{1}{4\omega_s}\tau c_0\sin\psi_0,\ J_{14}=\frac{1}{4\omega_s}\tau c_0^2\cos\psi_0,$$

$$J_{21}=\frac{\sigma_1}{b_0}-\frac{9}{8\omega_s}\beta_1b_0-\frac{25}{16\omega_s}\beta_2b_0^3-\frac{9}{4}\omega_s\delta b_0,$$

$$J_{22}=-\frac{1}{2\omega_sb_0}f_1\sin\varphi_0\cos(\alpha),$$

$$J_{23}=\frac{1}{2\omega_sb_0}\tau c_0\cos\psi_0,\ J_{24}=-\frac{1}{4\omega_sb_0}\tau c_0^2\sin\psi_0,$$

$$J_{31}=-\frac{1}{4\omega_c}\rho c_0\sin\psi_0,$$

$$J_{33}=-\xi\omega_c-\frac{1}{4\omega_c}\rho b_0\sin\psi_0,\ J_{34}=-\frac{1}{4\omega_c}\rho b_0c_0\cos\psi_0,$$

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$$J_{41}=-\frac{\sigma_1}{b_0}+\frac{9}{8\omega_s}\beta_1 b_0+\frac{25}{16\omega_s}\beta_2 b_0^3+\frac{9}{4}\omega_s\delta b_0-\frac{1}{2\omega_c}\rho\cos\psi_0,$$

$$J_{43}=-\frac{1}{c_0}(\sigma_1-\sigma_2)-\frac{1}{2b_0\omega_s}\tau c_0\cos\psi_0-\frac{1}{2c_0\omega_c}\rho b_0\cos\psi_0,$$

$$J_{44}=\frac{1}{2\omega_c}\rho b_0\sin\psi_0+\frac{1}{4b_0\omega_s}\tau c_0^2\sin\psi_0.$$

The stability of the non-trivial solution is investigated by evaluating the eigenvalues of the Jacobian matrix of equations (57) - (60)

$$\begin{vmatrix} J_{11}-\lambda & J_{12} & J_{13} & J_{14} \\ J_{21} & J_{22}-\lambda & J_{23} & J_{24} \\ J_{31} & 0 & J_{33}-\lambda & J_{34} \\ J_{41} & -J_{22} & J_{43} & J_{44}-\lambda \end{vmatrix}=0,$$

which gives

$$\lambda^4+\eta_1\lambda^3+\eta_2\lambda^2+\eta_3\lambda+\eta_4=0. \quad (61)$$

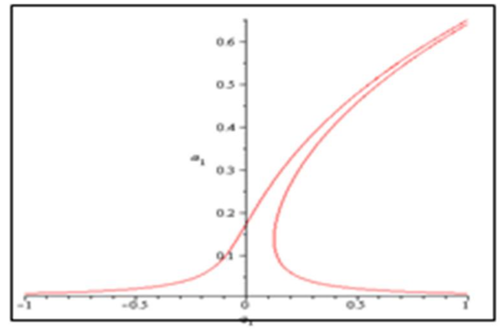
The non-trivial solution is stable if

$$\eta_1>0,\ \eta_1\eta_2-\eta_3>0,\ \eta_3(\eta_1\eta_2-\eta_3)-\eta_1^2\eta_4>0,\ \eta_4>0.$$

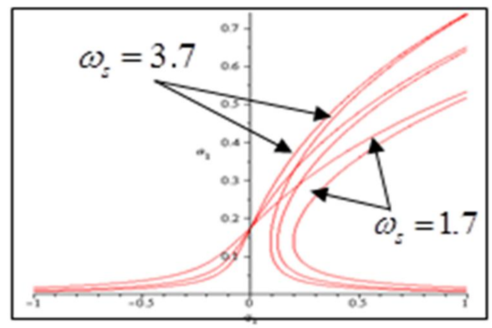
6 THEORETICAL FREQUENCY RESPONSE SOLUTION

The resonant frequency response equations of the main system (39) with NS controller (40) are solved numerically. The results are shown in Figs. (3) and (4) which represents the variation of the steady state amplitudes $a_{1,2}$ against the detuning parameter $\sigma_{1,2}$, respectively, for different values of the other parameters. Fig. (3) shows the theoretical frequency response curves of the main system to primary resonance case. It can be noted from Figs. (3b-3d) and (3g) that steady state amplitude increases as each of the natural frequency ω_s, the linear damping coefficient μ_1 and the nonlinear coefficients β_1 and δ decrease. The increase in the quintic nonlinear parameter β_2 bends the frequency response curves to the right with trivial effect on the steady state amplitude as shown in Fig. (3e). Fig. (3f) indicates that as the excitation force amplitude f increases, the branches of the response curves diverge away and the amplitude increases. The effect of the gain is shown in Fig. (3h).

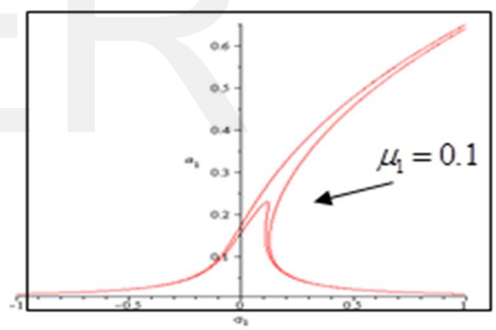
Fig. (4) illustrates the resonant frequency response curves of the NS controller to subharmonic internal resonance for various parameters. Each figure consists of two curves that either diverge away when the gain ρ and the steady state amplitude of the plant increase, Fig.(4b, 4f). Or, they converge to each others as the natural frequency ω_c and the linear damping ζ are decreased as shown in Fig.(4c, 4d). The curves in Fig. (4e) are shifted to the right as the detuning parameter σ_1 increases.



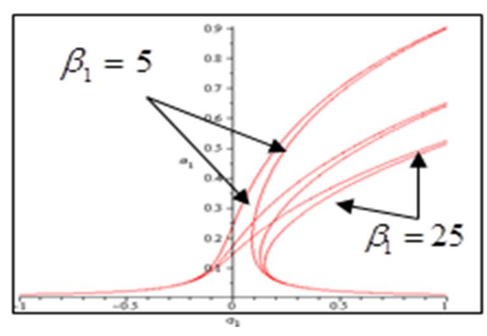
(a) Basic case



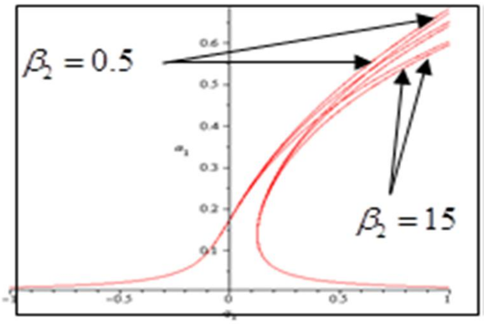
(b) The natural frequency



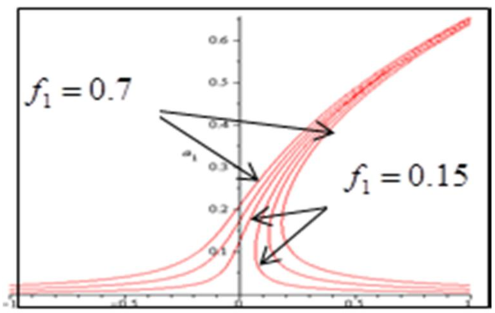
(c) The damping coefficient



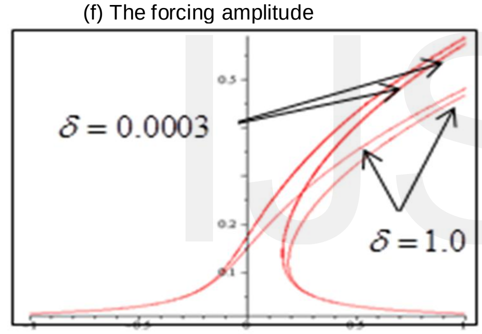
(d) The cubic nonlinear coefficient



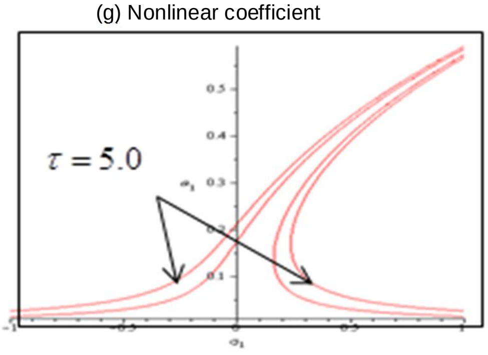
(e) The quintic nonlinear coefficient



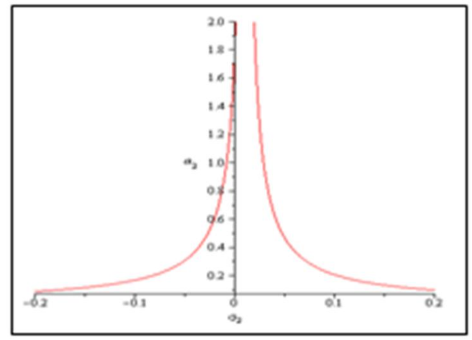
(f) The forcing amplitude



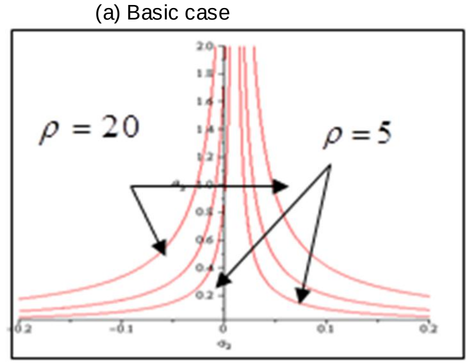
(g) Nonlinear coefficient



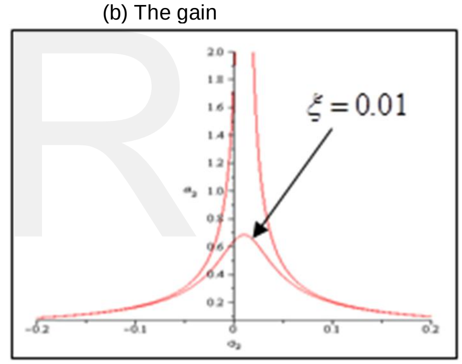
(h) The gain



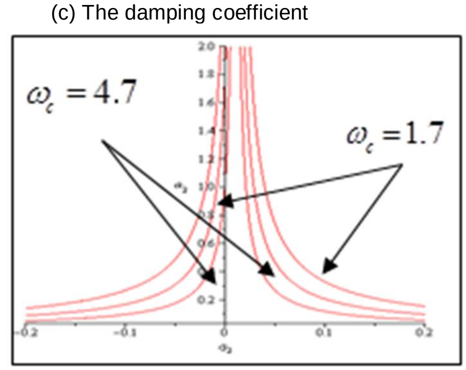
(a) Basic case



(b) The gain



(c) The damping coefficient



(d) The natural frequency

Fig. 3 Theoretical resonant frequency response curves of the plant when:
$\omega_s = 2.7,\ \beta_1 = 15.0,\ \delta = 0.03,\ \mu_1 = 0.0005,\ \beta_2 = 5.0,$
$f_1 = 0.4,\ \alpha = 30^0,\ \tau = 0.1.$

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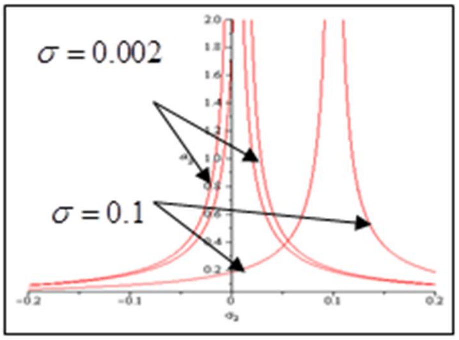
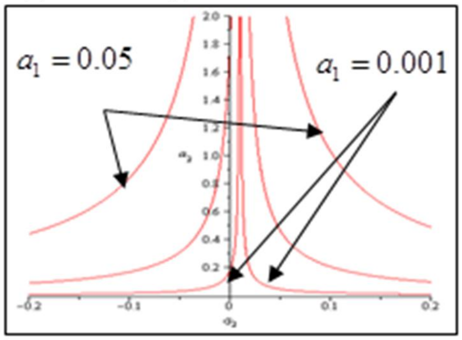


(e) The detuning parameter



(f) The steady state amplitude of the plant

Fig. 4 Theoretical resonant frequency response curves of the (NS) controller when: $\omega_c = 2.7,\ \xi = 0.0001,\ \sigma = 0.01,\ a_1 = 0.01,\ \rho = 10.0.$

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7 Conclusions

The control and stability of a nonlinear differential equation representing the one-degree-of-freedom nonlinear inclined beam are studied. The inclined beam has cubic and quintic nonlinearities subjected to external and parametric excitation forces. Two controller techniques have been applied to the inclined beam system under different resonance conditions and the results of numerical Runge-Kutta integration show that NS controller is the most effective. The analytical solutions of the plant with NS controller are obtained applying the multiple scales perturbation technique. The stability of the coupled system is investigated applying the frequency response equation for various values of the parameters of the plant and NS controller. In addition, a good criterion of both stability and chaos is the phase plane.

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